

Growth in Overlapping Generation Economies with Non-Renewable Resources*

Betty Agnani, María-José Gutiérrez and Amaia Iza
DFAEH - University of the Basque Country[†]

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[†]Correspondence to Universidad del País Vasco, Avda. Lehendakari Aguirre 83, 48015 Bilbao (Spain). Fax: 34-946013774. e-mail: jedaggab@xa.bs.ehu.es (Agnani), jepguhum@bs.ehu.es (Gutiérrez) and jepiz-paa@bs.ehu.es (Iza).

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ABSTRACT: Feasibility of positive steady-state growth in overlapping generation (OLG) economies that use non-renewable resources as essential inputs in the production process is analyzed. The model we use is, in essence, that of Diamond [9] with non-renewable resources and exogenous technological progress. The main finding is that having a high enough labor share is a necessary condition for the economy to exhibit positive steady-state growth rate. This condition does not need to be satisfied in infinitely-lived agent economies. The reason is that although technological progress is introduced exogenously, in the OLG economy, the growth rate of the economy depends among others on capital accumulation, which requires savings paid out of wage income. We also show that the unique balanced growth path is efficient in the Pareto sense, as expected.

Key Words: overlapping generations, exogenous technological progress, non-renewable resources, balanced growth path.

JEL classification: O13, O40, Q32

1 Introduction

The pessimistic Malthusian view of the Club of Rome that economic growth is constrained by natural resource scarcity was overcome by neoclassical economics in the 1970s. Natural resources have been introduced in a neoclassical growth framework as an essential factor of production and the results are dramatically in contrast with negative predictions. Economies may grow in spite of the scarcity of natural resources if there is a continuous flow of (exogenous) technical progress (Stiglitz [36], Solow [34], Dasgupta and Heal [8])¹. In particular, Stiglitz [36] analyzes the sustainability of a long-run positive growth rate in a model à la Solow-Swan with exhaustible resources as essential inputs and with exogenous technological progress and savings. He finds that the stationary depletion rate of the non renewable resource is endogenously given by the savings rate. Cass and Mitra [7] analyze technological possibilities, allowing scale indivisibilities or other types of nonconvexity, under which continually increasing and asymptotically unbounded consumption is feasible in the presence of exhaustible resources. In our paper we take the neoclassical production function with exogenous technological progress as in Stiglitz [36], and analyze the sustainability of a long-run positive growth rate in a model à la Diamond.

With the birth of endogenous growth models, the sustainability of positive long-run growth rate with exhaustible resources has also been analyzed. This new approach consists of analyzing the growth properties in infinitely-lived agents (ILA), in the spirit of Cass-Koopmans-Ramsey models. Among others, Aghion and Howitt [1], Barbier [5] and Nili [29] study the sustainability of positive long-run growth paths in economies whose engine of growth is determined by the resources devoted to the R&D sector. They focus mainly on characterizing the optimal solutions. Scholz and Ziemes [33] and Grimaud and Rouge [15] extend this analysis by studying the market equilibrium in this kind of economy. They

¹Good surveys of the contribution of neoclassical models to the natural resource economy can be found in Krautkraemer [23] and Neumayer [28].

focus on stability properties and prove that market equilibrium does not coincide with the optimal allocation because of distortions in the economy.

In contrast with ILA literature, we analyze the feasibility of long-run growth in overlapping generations (OLG) economies with finite-lived agents that use non-renewable resources as essential inputs in the production process. The model we use is, in essence, that of Diamond [9] with natural non-renewable resources and exogenous technological progress. As in Diamond [9] our framework assumes that agents are not altruistic enough and, consequently, do not receive any bequest. We think that the OLG framework is more appropriate for studying the sustainability of long-run growth with exhaustible resources than ILA models for several reasons. First, in ILA models all future impacts are treated as if they happened to current agents, ignoring the fact that society is composed of mortal individuals of different generations whose actions have consequences that outlive them. Authors such as Solow [35] and Padilla [31] consider that it is necessary to capture these intergenerational aspects in environment and natural resource economic analysis. Second, there exists strong empirical evidence against the idea that members of extended families are altruistically linked in the way that ILA models assume (see Altonji *et al.* [2]). And third, in analyzing natural resource management we must take into account, as Koskela *et al.* [22] point out, that natural resources may act as stores of values between different generations; therefore it is not clear that the properties of the ILA model are robust in an OLG framework.

As we know an OLG economy may contract, even in economies without exhaustible resources, when savings are insufficient to pay for an increasing capital stock, as already shown by Galor and Ryder [11], Jones and Manuelli [18], [19] and Fuster [10]. On the other hand, Gerlagh and Van der Zwaan [14] state that if environmental resources become a part of private assets, and if the total value of these assets becomes large compared with savings from revenues earned when young, this may cause the economy to contract. Furthermore,

these authors claim that the possibility of economic decline may be the unintended result of natural resource privatization. In particular, they state that if instead of making the usual assumption that the initial old generation is the owner of the total amount of the natural resources of the economy, its ownership could be shared by all generations, the equilibrium solution would be one of growing equilibrium instead of contracting equilibrium. Along the same lines, Gerlagh and Keyzer [13] analyze sustainability in an OLG economy with exhaustible resources, which are not essential for production but have amenity value and they are the only source of repository of savings, since there is no physical capital in the economy. The authors find that the economy would not contract under a *trust fund* policy. However, neither of these papers analyzes the possibility of positive balanced growth paths in economies with exhaustible resources, which are essential for production, in an OLG setting. Our work can be seen as an extension of Olson and Knapp [30] for the case in which long-run exogenous growth is considered. Olson and Knapp [30] fully characterize competitive allocations of economies endowed with exhaustible resources which are essential for production in an OLG framework where the possibility of long-run growth is not analyzed.² Our main finding is that since the steady-state growth rate of the economy depends on the economy's parameters, although technological progress is introduced exogenously, (see, e.g. Stiglitz [36]), our OLG setting imposes the same additional conditions for positive steady-state growth as in standard endogenous growth models without natural resources in contrast to ILA models. In particular we find, as Jones and Manuelli [18] [19] and Fuster [10] do in standard OLG endogenous growth models, that the labor share has to be high enough for a positive steady-state growth rate to exist.

Finally, we solve the Ramsey optimal control problem to show that the unique balanced growth path is efficient in the Pareto sense. This result is not surprising giving that in our economy the conditions mentioned by Rhee [32] or Gerlagh and Keyzer [13], in or-

²The OLG framework has also been used to model economies endowed with renewable resources. See, among others, Koskela *et al.* [22], Amacher *et al.* [3] and Mourmouras [27].

der for an economy with non-reproducible resources to be dynamic efficient are satisfied.

Both these previous papers apply a more direct approach as shown in Geanakoplos and Polemarchakis [12], for a pure exchange economy.³

The paper is organized in the following way. Section 2 presents the model. The competitive equilibrium is derived in Section 3. Section 4 characterizes the balanced growth path associated with the competitive equilibrium. Section 5 focus on optimal solutions and analyzes the optimality of the balanced growth path. Conclusions are presented in Section 6.

2 The model

The general structure of our model develops the basic two-period OLG model (Diamond [9]) with one good production in an economy endowed with exhaustible resources and physical capital. Two generations cohabit in any period of time t : young and old. Every generation consists of N_t new individual agents or families who live for two periods (finite lifetimes). Population grows at a constant rate n , i.e. $N_t = N_{t-1} (1 + n)$ and the number of people living in period t is $N_t + N_{t-1}$. We model a grandfathering economy without altruism in which the initial endowments of exhaustible resources and physical capital belongs to the first generation of elderly agents and agents do not care about the welfare of future generations.⁴ Initial elderly agents sell natural resources to their successors to provide for old age and firms in order to produce, and so on in every period.

Consumers

All individual agents have rational expectations and are identical except for their

³As Kemp and Van Long [20] point out, in an OLG economy the resource plays the double role of repository of savings and source of productive inputs. The decline of the non-renewable resource may force its price so high that it ceases to perform its second function since its marginal product is not high enough. This is suboptimal.

However, if the resource is essential for production, this reasoning is not valid. This is shown by McCallum [32], who argue that capital over-accumulation is impossible if an economy includes an asset that is productive and non-reproducible, and by Rhee [26], among others, who shows that this argument is valid under certain conditions on the production function.

⁴If one-side altruism is considered instead, the equilibrium is as in ILA models.

ages. As usual in growth literature, we are interested in economies for which balanced growth paths exist; therefore, we need to assume consumer's preferences with constant elasticity of intertemporal substitution (King and Rebelo [21]). In particular we consider that the preferences of the representative agent born at period t are represented by a constant unitarian elasticity of intertemporal substitution function,

$$u(c_{1t}, c_{2,t+1}) = \ln c_{1t} + \frac{1}{1+\theta} \ln c_{2,t+1},$$

where c_{1t} and $c_{2,t+1}$ represent consumption in young and old age, respectively; $\theta \geq 0$ is the subjective discount rate of the agent.

Each agent born at period t is endowed with one unit of labor when she is young and supplies it to firms inelastically. She receives the wage, w_t , which is used to consume in the first period, c_{1t} , to buy ownership rights for resource stock, m_t , and to save as physical capital, s_t . The price of the ownership right of one unit of the exhaustible resource in terms of the consumption good is given by p_t .

When the agent is old, at period $t+1$, her income comes from two different sources. On the one hand, firms pay $(1+r_{t+1})s_t$ to the agent for renting her capital stock. On the other hand, the agent receives the revenue for selling her natural resources, $p_{t+1}m_t$.

Therefore, the representative agent born at period t maximizes her utility function with respect to young and old consumption and ownership of resources taking prices as given. This problem can be written as follows, $\forall t = 1, 2, \dots$

$$\begin{aligned} \underset{\{c_{1,t}, c_{2,t+1}, s_t, m_t\}}{\text{Max}} \quad & \ln(c_{1,t}) + \frac{1}{1+\theta} \ln(c_{2,t+1}), \\ \text{s.t.} \quad & \end{aligned}$$

$$c_{1,t} + p_t m_t + s_t = w_t, \quad (1)$$

$$c_{2,t+1} = (1+r_{t+1})s_t + p_{t+1}m_t. \quad (2)$$

First order conditions for the consumer's optimization problem take the form

$$(1+\theta) \frac{c_{2,t+1}}{c_{1,t}} = 1 + r_{t+1}, \quad (3)$$

$$1 + r_{t+1} = \frac{p_{t+1}}{p_t}. \quad (4)$$

Equation (3) indicates that each consumer equates the marginal rate of substitution between current and future consumption to their relative prices, or marginal rate of transformation. Equation (4) is the standard arbitrage condition that characterizes the optimal investment between the two forms of saving such that the marginal returns on both are equal. In other words, the marginal rate of investing in the exhaustible resource, p_{t+1}/p_t , must be equal to the marginal rate of investing in physical capital, $1 + r_{t+1}$.⁵

Firms

There is an indeterminate number of competitive firms producing a homogeneous good that can be consumed or invested in the form of physical capital. All firms share the same production technology and use as inputs labor, N_t , tangible capital, K_t and exhaustible resource, X_t , given a technology level, B_t , at each period t . We assume perfect substitutability between all inputs, in particular, we consider an aggregate production function for the final output given by $Y_t = B_t K_t^\alpha N_t^\beta X_t^\nu$, that presents constant returns to scale with respect to all inputs for a given technology level, i.e., $\alpha + \beta + \nu = 1$.

Since Y_t is homogeneous of degree one with respect to private factors, the production function can be expressed in per-worker terms as

$$y_t = B_t k_t^\alpha x_t^\nu, \quad (5)$$

where $y_t = Y_t/N_t$, $k_t = K_t/N_t$ and $x_t = X_t/N_t$.

We assume that technology grows at an exogenous constant rate b . Therefore, the

⁵This arbitrage condition satisfies the well-known Hotelling rule of optimal resource extraction for exhaustible resources in partial equilibrium models, under assumption of costless extraction (Hotelling [17]).

evolution of technology can be expressed as

$$B_{t+1} = (1 + b)B_t. \quad (6)$$

Firms hire labor, capital and exhaustible resource to maximize profits taking prices, interest rate and technology as given. Therefore, the representative firm's problem can be settled as follows

$$\begin{aligned} \max_{\{Y_t, K_t, X_t\}_{t=0}^{\infty}} \quad & \Pi_t = Y_t - w_t N_t - (r_t + \delta) K_t - p_t X_t, \\ \text{s.t.} \quad & Y_t = B_t K_t^\alpha N_t^\beta X_t^\nu, \end{aligned}$$

where $0 \leq \delta \leq 1$ is the capital depreciation rate.

In the case of an interior solution, the first-order conditions for the firm's optimization problem are given, in per-worker terms, by

$$\alpha B_t k_t^{\alpha-1} x_t^\nu = r_t + \delta, \quad (7)$$

$$\nu B_t k_t^\alpha x_t^{\nu-1} = p_t, \quad (8)$$

$$\beta B_t k_t^\alpha x_t^\nu = w_t, \quad (9)$$

which indicates that firms hire capital, exhaustible resources and work until their marginal products equal their factor prices.

Exhaustible resources

The economy is initially endowed with a positive amount of the exhaustible resource, M_{-1} , which belongs to the first generation of elderly agents. At each period t , the total stock of the exhaustible resource, $M_t = m_t N_t$, is determined by the past resource stock less current resources used for production, i.e., $M_t = M_{t-1} - X_t$.

The depletion rate of the exhaustible resource is defined as⁶ $\tau_t = X_t/M_{t-1}$, which can be written in per worker terms as

$$\tau_t = (1 + n) \frac{x_t}{m_{t-1}}. \quad (10)$$

⁶Note that $0 < \tau_t < 1$. As Kula [24] points out, the rate of use (or growth) of the renewable resource is rephrased in the context of exhaustible resource to the rate of depletion.

Taking this into account, the dynamics of the exhaustible resource can be expressed, in per worker terms, as

$$m_t = \frac{1 - \tau_t}{1 + n} m_{t-1}. \quad (11)$$

The non-renewability of the resource implies that the amount of resource used by all generations cannot be greater than the initial stock, M_{-1} . In per worker terms, this can be expressed as

$$m_{-1} \geq \sum_{t=0}^{\infty} \tau_t (1 + n)^t m_{t-1}.$$

Taking into account the dynamics of the resource, (11), this would imply the following *exhaustibility condition*

$$1 \geq \sum_{t=0}^{\infty} \tau_t \prod_{j=1}^t (1 - \tau_{j-1}).$$

When the depletion rate is constant in time, this condition holds with equality, i.e. the economy depletes the resource in the long run.⁷

3 Competitive equilibrium

A dynamic competitive equilibrium is a sequence of allocations $\{y_t, k_{t+1}, s_t, \tau_t, c_{1t}, c_{2t}, m_t, x_t, B_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, p_t\}_{t=0}^{\infty}$ given initial values for the state variables k_0, M_{-1}, B_0 and the law of motion for B_t , such that consumers maximize utility, firms maximize profits and markets clear.

Market clearing

As the economy produces a unique good that is either invested in physical capital or consumed, the good market clearing condition is given by

$$s_t = k_{t+1} (1 + n). \quad (12)$$

This condition indicates that savings from the young generations determine the stock of physical capital of the next period, as in Diamond [9].

⁷We show in section 4 that the balanced growth path equilibrium is characterized by a constant depletion rate and therefore for a total exhaustion of the resource in the long run.

The resource market clearing condition is given by equation (11), which implies that non renewable resources supplied by old agents must be equal to firms' and young agents' demand.

Definition 1 *A competitive equilibrium of this economy is an infinite sequence of allocations $\{y_t, k_{t+1}, s_t, \tau_t, c_{1t}, c_{2t}, m_t, x_t, B_{t+1}\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, p_t\}_{t=0}^{\infty}$, given the initial positive conditions⁸, $k_0, M_{-1}, B_0 > 0$ and the law of motion for B_t , such that consumers and firms maximize their objective functions taking prices as given. The endogenous sequence for prices $\{w_t, r_t, p_t\}_{t=0}^{\infty}$ is obtained such that all markets clear. In other words, a competitive equilibrium is a solution of the non-linear system, (1)-(12).*

4 Balanced Growth Path

We focus on balanced growth paths, i.e. paths which are characterized by constant growth rates of all variables. The reason for this choice is that balanced growth paths are the only kind of path that can generate long-run positive growth in the economy⁹. In particular we analyze those paths defined as follows:

Definition 2 *A balanced growth path is a competitive equilibrium where all variables grow at a constant rate.*

Let us define γ_z as the ratio z_{t+1}/z_t in the balanced growth path for all variables. With this notation, $\gamma_z - 1$ represents the growth rate of variable z . Given the dynamics of the exhaustible resource, equation (11), the depletion rate must be constant on the balanced growth path because m_t decreases at a constant rate.¹⁰ This means that a balanced growth

⁸Since capital, resources and technology are essential for production, k_0 , M_{-1} and B_0 must be positive. Otherwise young consumers of the initial generation would have no income and consumption would remain zero forever.

⁹We do not focus on dynamics from points outside the balanced growth path. The dynamics in ILA models has been analyzed by Nile [29], Scholz and Ziemens [33] and Barbier [5].

¹⁰On the balanced growth path the interest rate will be also constant as can be easily seen from condition (4), taking into account that the growth of p_t is constant.

path is characterized by a positive flow of extraction of resources that gets ever smaller and smaller. Such a pattern is called *asymptotic depletion*¹¹ and we define $\bar{\tau} = \tau_{t+1} = \tau_t$.

The following lemma states conditions that any balanced growth path of this OLG economy must satisfy.

Lemma 1 *Any balanced growth path of this economy is characterized by*

i) *A stationary depletion rate, $\bar{\tau}$, that is the solution of the following non linear equation*

$$\frac{\gamma(1+n)}{1-\bar{\tau}} = \frac{\alpha(1+n)(2+\theta)\gamma}{\beta - (2+\theta)\nu(1-\bar{\tau})/\bar{\tau}} + (1-\delta),$$

where $\gamma = (1+b)^{\frac{1}{1-\alpha}} \left(\frac{1-\bar{\tau}}{1+n}\right)^{\frac{\nu}{1-\alpha}}$.

ii) *The following growth rates*

$$\gamma_y = \gamma_k = \gamma_{c_1} = \gamma_{c_2} = \gamma_s = \gamma_w = \gamma,$$

$$\gamma_b = (1+b),$$

$$\gamma_m = \gamma_x = \frac{1-\bar{\tau}}{1+n},$$

$$\gamma_p = \gamma \frac{1+n}{1-\bar{\tau}},$$

$$\gamma_r = 1.$$

Proof. See Appendix 1. ■

This lemma states that on the balanced growth path, income, physical capital, consumption, saving and wages grow at the same rate, $(\gamma - 1)$, which depends in a particular manner on all structural parameters of the economy. The price of the non-renewable resource grows at a larger rate than income, highlighting the fact that the resource is exhaustible. In fact, the value of the exhaustible resource, $p_t m_t$, grows at the same rate as physical capital and income, γ . Since the gross interest rate is the growth rate of the resource price (arbitrage condition (4)), the interest rate must be constant on the balanced

¹¹See Hartwick and Olewiler [16, page 35] for a discussion of this issue.

growth path. Observe that the resource stock and the resource used in the production process, M_t and X_t , decline over time. Moreover, if the growth rate of the economy is positive, i.e. if $\gamma > 1$, then the price of the exhaustible resource increases over time. Both results are standard in resource economic models.¹²

Notice that the stationary depletion rate, $\bar{\tau}$, is determined by a non-linear equation and depends on all parameters of the economy. This result contrasts with the standard ILA economies with non-renewable resources. In this type of model the stationary depletion rate depends exclusively on the subjective discount factor.¹³

The lemma also establishes an upper bound for the stationary depletion rate in order to guarantee positive growth in the economy. In particular, $\gamma > 1$ if the endogenous stationary depletion rate is such that $\bar{\tau}(\alpha, \beta, v, \theta, b, n) < 1 - (1 + b)^{-1/\nu} (1 + n)$.¹⁴ We call this threshold *growth upper bound (gub)*.

Lemma 1 imposes conditions that must be satisfied by any balanced growth path. However, up to now, there is no guarantee that this stationary depletion rate exists and is unique, or that it can let the economy grow. The following proposition characterizes the stationary depletion rate, $\bar{\tau}$, and the constant growth rate, γ .

Proposition 1 *In this OLG economy, the balanced growth path is characterized by the following factors:*

i) *The stationary depletion rate, $\bar{\tau}$, exists and is unique,*

ii) *The stationary depletion rate satisfies $\frac{(2+\theta)\nu}{\beta+(2+\theta)\nu} < \bar{\tau} < 1$,*

¹²In the basic Hotelling model the price of the resource increases over time. However in other contexts the resource path may present a different pattern. For instance when exploration of new reserves is considered the resource price path can be U-shaped (Livernois and Uhler [25]). Tahvonen and Salo [37] find that resource prices may decrease over time when studying transitions in economies with nonrenewable and renewable resources.

¹³This outcome stems from the fact that consumers' utility function is logarithmic where the elasticity of intertemporal substitution is one. The characterization of the balanced growth path for an ILA economy is available to readers upon request.

¹⁴It is easy to show that in an ILA economy this condition becomes $(1 + b) > [(1 + \theta)(1 + n)]^\nu$ because $\bar{\tau}^{ILA} = \theta / (1 + \theta)$.

iii) $\partial\gamma/\partial b > 0$, $\partial\gamma/\partial\theta < 0$ and $\partial\gamma/\partial\delta > 0$.

Proof. See Appendix 1. ■

This proposition guarantees that the stationary depletion rate exists and is unique under no additional conditions. As usual, economies with high technological parameters, b , have high growth rates. Moreover, if consumers become more impatient and prefer to consume more when they are young, then the constant growth rate decreases because the natural resource depletion rate goes up. By contrast, an increase of the physical capital depreciation rate, δ , makes physical capital relatively more expensive than natural resources. This implies that the economy becomes more conservative because agents save more in resources reducing the stationary depletion rate and, therefore, the economy constant growth rate increases.¹⁵

Proposition 1 establishes a lower bound for the stationary depletion rate: the balanced growth path is always characterized by a depletion rate which is higher than $\frac{(2+\theta)\nu}{\beta+(2+\theta)\nu}$. We call this threshold *existence lower bound (elb)*. This kind of lower bound does not appear in characterizing the stationary depletion rate in the ILA economies.

Combining the thresholds defined from Lemma 1 and Proposition 1 we can say that an OLG economy that exhibits positive constant growth satisfies $elb < \bar{\tau} < gub$, i.e.

$$elb \equiv \frac{(2 + \theta) \nu}{\beta + (2 + \theta) \nu} < \bar{\tau} < 1 - \frac{(1 + n)}{(1 + b)^{1/\nu}} \equiv gub.$$

This means that the economy may contract under several circumstances. The economy will contract if the relation between the parameters is such that $gub \leq elb$. There exist many combinations of the parameters b, n, θ, v , and β that may imply $gub \leq elb$. For instance, when the growth rate of population is too high compared to the technological level, the gub threshold may become very low such that $gub < elb < \bar{\tau}$. Also, the lower the labor share, β ,

¹⁵The proposition does not show how the growth rate changes with the production parameters, β, v and α . This is because a change in any of these parameters will necessarily be accompanied by changes in the others in order to keep the constant returns to scale assumption ($\beta + v + \alpha = 1$). Later on we illustrate this issue with a numerical exercise.

the easier it is to have $gub < elb$, and, in consequence, the OLG economy would converge to a zero production. It could even happen that with the same parameters b, n, θ and ν an ILA economy may show positive growth while an OLG economy contracts.¹⁶

This is not new in the literature of overlapping generations. Galor and Ryder [11] show that an OLG economy without natural resources may contract if certain conditions on the technology, preferences and the interaction between preferences and technology are satisfied. Fuster [10] and Jones and Manuelli [18] [19] show in a standard endogenous growth model, where all production factors but labor are endogenously reproducible, that a productive enough technology that generates positive long-run growth in an ILA economy may not be sufficient in an OLG economy, because young agents do not have enough income to buy an increasing stock of physical capital. Gerlagh and Van der Zwaan [14] link this feature to the value share of natural resources in production. In particular, they mention that if environmental resources become a part of private assets, and if the total value of these assets becomes large compared with savings from revenues earned when young, this may cause the economy to contract. The following proposition formally presents this result.

Proposition 2 *If the labor share is such that $\beta < \frac{(1+b)^{-1/v}(2+\theta)(1+n)\nu}{[1-(1+n)(1+b)^{-1/v}]}$, then this OLG economy contracts.*

Proof. The economy contracts whenever that $\tau < gub$ does not hold. In particular, the economy contracts if $gub < elb$. The result follows directly from this inequality. ■

The meaning of the lower bound for the labor share parameter in the OLG setting is clear. If the parameter β is not high enough there is not guarantee that each new generation will receive enough income (wage rate) to be able to buy an increasing capital stock and a decreasing stock of the exhaustible resource whose price is increasing, which are the characteristics of a growing economy.

¹⁶This is so because the labor share parameter, β , does not appear in the condition for an ILA economy to exhibit positive growth (see footnote 14).

Figure 1: OLG growth rates for any combination (α, β, v) that belong to the simplex

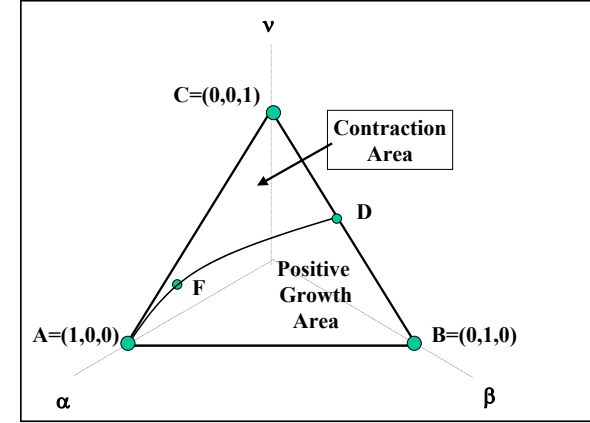


Figure 1 illustrates with a numerical example how the growth rate of an OLG economy changes for different combinations of productive parameters keeping the constant return to scale assumption, i.e. $\alpha + \beta + v = 1$.

This numerical example considers an economy where each period covers 25 years with the following annual parameter values: $n = 0.01$, $\delta = 0.027$, $\theta = 0.016$, $b = 0.028$. Each point in the triangle represents a combination (α, β, v) that belongs to the simplex that sums to one. Points A, B and C represent combinations $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$, respectively. Any point in the lines AB, BC, AC represent a combination in which $\nu = 0$, $\alpha = 0$ and $\beta = 0$, respectively. Two facts should be mentioned. First, any combination in the segment AB represents economies without natural resources, $\nu = 0$, and therefore always exhibits positive growth with $\gamma = (1 + b)^{1/(1-\alpha)}$. Second, any combination in the segment AC represents economies that always contract because the labor share is zero, $\beta = 0$, and therefore the condition presented in Proposition 2 holds. Point F represents an economy in which $\alpha = 0.875$, $\beta = 0.01$ and $v = 0.125$. Point D is an economy with $\alpha \simeq 0$, $\beta = 0.489$ and $v = 0.509$. We have shown numerically that combinations of (α, β, v) that are above

the curve formed by points A, F and D imply contraction of the economy; otherwise the economy exhibits positive long run growth.¹⁷ It is easy to verify that for these parameter values an ILA economy never contracts regardless of the production function parameters. On the other hand, usual values of factor participation in output such as $\alpha = 0.60$, $\beta = 0.35$ and $v = 0.05$, imply that in the long run this economy grows annually 6.2% and the remained resource is depleted 6% every period.

5 Optimality

Geanakoplos and Polemarchakis [12] points out that to prove dynamic efficiency in an OLG pure exchange economy it is sufficient to guarantee that the first generation owns a resource that, in every period, contributes to income with a strictly positive share bounded away from zero. Using the well-known Balasko-Shell condition (Balasko and Shell [4]), Rhee [32] shows that an OLG economy with land will be efficient in the Pareto sense if the income share of land does not vanish in the long run, or equivalently if the initial value of the resource is finite (see his proposition 1). Gerlagh and Keyzer [13] also applies this condition (see their lemma 1, proposition 2) to show that the equilibria are dynamically efficient in an OLG economy with natural resources, which are valuable but not essential for the production.¹⁸

Another approach to show that the balanced growth path is dynamically efficient is through the analysis of the allocation chosen in a Ramsey Optimal control economy. Formally, a social planner solves the following problem,

$$\begin{aligned} \underset{\{c_{1,t}, c_{2,t}, m_{t+1}, k_{t+1}, \tau_t\}_{t=0}^{\infty}}{\text{Max}} \quad & \frac{1}{1+\theta} \ln(c_{2,0}) + \sum_{t=0}^{\infty} \frac{1}{(1+R)^{t+1}} \left[\ln(c_{1,t}) + \frac{1}{1+\theta} \ln(c_{2,t+1}) \right], \end{aligned} \quad (13)$$

¹⁷We have shown numerically that the curve in the segment FD is almost linear. However, the model does not behave numerically well for combinations very close to point A. In particular, we have problem to solve the model numerically for values of $\beta < 0.01$.

¹⁸In our model, the exhaustible resource's income share is a positive constant, v . Therefore, as Rhee [32] shows, the Balasko-Shell condition always holds and the balanced growth path is always optimal. The complete proof is available upon request.

$$s.t. \begin{cases} y_t = c_{1,t} + \frac{c_{2,t}}{(1+n)} + (1+n)k_{t+1} - (1-\delta)k_t, \\ m_{-1} = \sum_{t=0}^{\infty} \tau_t (1+n)^t m_{t-1}, \\ m_t = \frac{1-\tau_t}{1+n} m_{t-1}, \\ x_t = \frac{\tau_t}{1+n} m_{t-1}, \\ B_{t+1} = (1+b)B_t, \\ y_t = B_t k_t^\alpha x_t^v, \\ k_0, B_0, m_{-1} > 0 \text{ are given,} \end{cases}$$

where $R > 0$ is the subjective discount rate of the planner.¹⁹ The first restriction of problem (13) is the good constraint of the economy in period t and it means that the total supply of goods provides for consumption by young and old individuals and for the next period's capital stock. The second restriction represents the exhaustibility condition and tell us that the amount of resource assigned to all generations must be equal to the initial stock of resource. The third and fourth restrictions show the dynamics of the stock of exhaustible resource and the technology level, respectively. The fifth restriction is the production function.

After some algebra, the first order condition of the planner's optimization problem can be expressed as

$$\frac{(1+\theta)c_{2,t}}{(1+R)c_{1,t}} = (1+n), \quad (14)$$

$$(1+\theta) \frac{c_{2,t+1}}{c_{1,t}} = \alpha B_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^v + (1-\delta), \quad (15)$$

$$\nu B_{t+1} k_{t+1}^\alpha x_{t+1}^{v-1} = [\alpha B_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^v + (1-\delta)] [\nu B_t k_t^\alpha x_t^{v-1}], \quad (16)$$

$$\lim_{t \rightarrow \infty} \left(\frac{1}{1+R} \right)^t \frac{k_{t+1}}{c_t} = 0. \quad (17)$$

Equation (14) represents the *intergenerational optimal consumption allocation*, and states that in any period the marginal rate of substitution between consumption of the young and consumption of the old must be equal to the rate of transformation of the economy, $(1+n)$, from the point of view of the central planner.

¹⁹Note that if R is non-positive the sum of the utilities does not converge. However, we consider the borderline case, $R = 0$, because it is possible in this case to discuss optimality using the *overtaking criterion* (Burmeister [6]), which essentially states that path A overtakes path B if there exists a finite t^* such that the present value of the future utilities associated with path A up to time t^* exceeds that associated with path B up to t^* , and that inequality remains in the same direction for all $t > t^*$. A path is optimal if it overtakes all other paths.

Equation (15) is the *intragenerational optimal consumption allocation* and indicates that consumption for each generation is chosen such that the marginal rate of substitution between present and future consumption of any agent must be equal to the net marginal product of physical capital. Observe that this optimal condition is the same one that consumers would derive from maximization of utility in a competitive equilibrium whenever the interest rate, r_t , is equal to the net marginal product of capital, $f_k - \delta$.

Equation (16) represents the *intertemporal resource allocation* condition. Observe that this optimal condition is also satisfied by the competitive equilibrium (see equation (4) once the firms' first order conditions (7) and (8) have been substituted) which indicates that the decrease in the stock of the exhaustible resource must imply an implicit return equal to the return of the physical capital. Finally, equation (17) is the transversality condition for capital.

The optimal solution characterization is summarized in the following definition.

Definition 3 *An optimal solution in this OLG economy is an infinite sequence of allocations $\{y_t, c_{1,t}, c_{2,t}, k_{t+1}, m_t, \tau_t, B_{t+1}\}_{t=0}^{\infty}$ which solves the planner's problem, given the initial conditions $k_0, m_{-1}, B_0 > 0$. In other words, an optimal sequence is a solution to the non-linear system (14)-(17) jointly with the restrictions of problem (13), given the above initial conditions.*

As we are interested in analyzing the optimality of the competitive balanced growth equilibrium path of this economy, we focus on the optimal solution in which growth is constant, i.e. in which all variables grow at a constant rate and the depletion rate is constant. Let us define $\tilde{\gamma}_z$ as the ratio z_{t+1}/z_t , on the optimal path with constant growth, for all variables. For the depletion rate, we define $\tilde{\tau} = \tau_{t+1} = \tau_t$. The following lemma characterizes the optimal balanced growth path sequence.

Lemma 2 *For a given $R > 0$, the optimal balanced growth path sequence can be represented*

by a vector $\{\tilde{\gamma}_y, \tilde{\gamma}_k, \tilde{\gamma}_m, \tilde{\gamma}_b, \tilde{\gamma}_{c_1}, \tilde{\gamma}_{c_2}, \tilde{\tau}\}$ such that,

$$\begin{aligned}\tilde{\gamma}_y &= \tilde{\gamma}_k = \tilde{\gamma}_{c_1} = \tilde{\gamma}_{c_2} = \tilde{\gamma}, \\ \tilde{\gamma}_m &= \frac{1}{(1+R)(1+n)}, \\ \tilde{\gamma}_b &= 1+b, \\ \tilde{\tau} &= \frac{R}{1+R},\end{aligned}$$

where $\tilde{\gamma} = (1+b)^{\frac{1}{1-\alpha}} [(1+R)(1+n)]^{-\frac{\alpha}{1-\alpha}}$.

Proof. See Appendix 1. ■

Notice that optimal behavior implies that the lower the planner's discount rate is, the lower the depletion rate and the larger the growth rate are. This means that the more the central planner cares about future generations, the more conservative the economy is and the larger the growth rate reached is.²⁰

Proposition 3 *The competitive balanced growth path of this OLG economy is optimal in the Pareto sense.*

Proof.

It is immediately evident by comparing the competitive equilibrium characterization (lemma 1) and optimal balanced growth (lemma 2), that for any $\bar{\tau}$ equilibrium solution there will exist a value for $R^* > 0$ such that $\tilde{\tau} = \bar{\tau}$ and $\tilde{\gamma} = \gamma$. ■

6 Conclusions

In this paper we have analyzed the feasibility of the positive balanced-growth path in OLG economies that use exhaustible resources as essential inputs in the production process. The model we have studied is, in essence, that of Diamond [9] with non-renewable resources and exogenous technological progress.

²⁰It is easy to see that the balanced growth path of an ILA economy has the same characterization as the optimal balanced growth path but with $R = \theta$.

We find the same results as Stiglitz [36], who finds that the steady-state growth rate of the economy depends on the exogenous savings rate, in contrast to the standard Solow growth model: although technological progress is introduced exogenously, the steady-state growth rate of the economy depends on all parameters of the economy, as in standard endogenous growth models.

Furthermore, as in standard endogenous growth models without natural resources, our model imposes the same additional conditions for positive steady-state growth, in contrast to ILA models. In particular, we find, as in Jones and Manuelli [18] [19], Fuster [10] or Galor and Ryder [11], that the labor share has to be high enough for there to be a positive steady-state growth rate. This additional condition guarantees that each new generation will receive enough labor income to be able to buy the increasing capital stock created by an economy with positive long run growth and the stock of the exhaustible resources whose value also increases.

Finally, we have analyzed welfare issues characterizing the optimal equilibria. As in previous papers we have found that the unique competitive balanced growth path is optimal in the Pareto sense. Rhee [32], Gerlagh and Keyzer [13], among others, show that OLG economies with natural resources can be efficient in the Pareto sense if the initial value of the resource is finite.

Appendix

Proof of lemma 1 :

Any competitive equilibrium is an infinite sequence of allocations $\{k_{t+1}, s_t, B_{t+1}, \tau_t, y_t, c_{1t}, c_{2t}, m_t, x_t\}_{t=0}^{\infty}$ and prices $\{w_t, r_t, p_t\}_{t=0}^{\infty}$ that solves the non-linear system, (1)-(12).

Proof of $\gamma_m = (1 - \bar{\tau}) / (1 + n)$:

Straightforward from valuation of equation (11) on the balanced growth path.

Proof of $\gamma_x = \gamma_m$:

Straightforward from taking the ratio of this equation (10) in period $t + 1$ and t , and valuing on the balance growth path.

Proof of $\gamma_b = (1 + b)$:

By definition of the technology evolution, (6).

Proof of $\gamma_k = \gamma$:

Combining equations (4), (7) and (8) the following intertemporal resource condition is obtained

$$\frac{B_{t+1}k_{t+1}^{\alpha}x_{t+1}^{v-1}}{B_t k_t^{\alpha} x_t^{v-1}} = (1 - \delta) + \alpha B_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^v. \quad (18)$$

Evaluating this expression in the balanced growth path and reordering,

$$\gamma_b \gamma_k^{\alpha} \gamma_x^{v-1} - (1 - \delta) = \alpha B_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^v. \quad (19)$$

Taking the ratio of this expression in period $t + 1$ and t , and valuing on the balanced growth path,

$$1 = \gamma_b (\gamma_k)^{\alpha-1} (\gamma_x)^v. \quad (20)$$

Substituting γ_b and γ_x , we obtain

$$\gamma_k = (1 + b)^{1/1-\alpha} \left(\frac{1 - \bar{\tau}}{1 + n} \right)^{v/1-\alpha} \equiv \gamma.$$

Proof of $\gamma_y = \gamma_k$:

Taking the ratio of the production function in period $t+1$ and t , and valuing on the balanced growth path we get $\gamma_y = \gamma_b (\gamma_k)^\alpha (\gamma_x)^v$. Taking (20) into account here, we get $\gamma_y = \gamma_k$.

Proof of $\gamma_s = \gamma_k$:

Combining equations (1) and (2), we obtain the saving function

$$s_t = \frac{w_t}{2+\theta} - p_t m_t.$$

Substituting the expressions (8), (9) and (10), the following expression is obtained

$$s_t = B_t k_t^\alpha x_t^v \left[\frac{\beta}{(2+\theta)} - \nu \frac{1-\tau_t}{\tau_t} \right].$$

Taking the ratio of this expression in period $t+1$ and t , and evaluating on the balanced growth path we get $\gamma_s = \gamma_b (\gamma_k)^\alpha (\gamma_x)^v = \gamma_y = \gamma_k$.

Proof of $\gamma_{c_1} = \gamma_{c_2} = \gamma_c$:

Taking the ratio of expression (3) in period $t+1$ and t , and considering equations (4),

and (8) we get

$$\frac{c_{2,t+1}}{c_{2,t}} \frac{c_{1,t-1}}{c_{1,t}} = \frac{B_{t+1} k_{t+1}^\alpha x_{t+1}^{v-1}}{B_t k_t^\alpha x_t^{v-1}} \frac{B_{t-1} x_{t-1}^{v-1}}{B_t k_t^\alpha x_t^{v-1}}.$$

And evaluating this on the balanced growth path, $\gamma_{c_1} = \gamma_{c_2} = \gamma_c$.

Proof of $\gamma_c = \gamma_k$:

Substituting equations (7), (8), (10), and (12) in (2), we get

$$c_{2,t+1} - (1-\delta)(1+n)k_{t+1} = B_{t+1} k_{t+1}^\alpha x_{t+1}^v \left[\alpha(1+n) + v \frac{1+n}{\tau_{t+1}} \right].$$

Taking the ratio of this expression in period $t+1$ and t , and evaluating on the balanced growth path we get

$$\frac{c_{2,t+1} - (1-\delta)(1+n)k_{t+1}}{c_{2,t} - (1-\delta)(1+n)k_t} = \gamma_b (\gamma_k)^\alpha (\gamma_x)^v = \gamma_k,$$

$$\frac{\gamma_{c_2}}{\gamma_k} c_{2,t} - (1-\delta)(1+n)k_t = c_{2,t} - (1-\delta)(1+n)k_t,$$

which implies that $\gamma_k = \gamma_{c_2} = \gamma_c$.

Proof of $\gamma_w = \gamma_k$:

Taking the ratio of expression (9) in period $t+1$ and t , and evaluating on the balanced growth path, we get $\gamma_w = \gamma_b (\gamma_k)^\alpha (\gamma_x)^v = \gamma_y = \gamma_k$.

Proof of $\gamma_r = 1$:

Taking the ratio of expression (7) in period $t+1$ and t , and evaluating on the balanced growth path,

$$\bar{r} = \frac{\alpha B_{t+1} k_{t+1}^{\alpha-1} x_{t+1}^v - \delta}{\alpha B_t k_t^{\alpha-1} x_t^v - \delta},$$

which is equal to one if expression (19) is considered.

Proof of $\gamma_p = \gamma/\gamma_m$:

Taking the ratio of expression (8) in period $t+1$ and t , and evaluating on the balanced growth path, we get $\gamma_p = \gamma_b (\gamma_k)^\alpha (\gamma_x)^{v-1} = \frac{\gamma_y}{\gamma_x} = \frac{\gamma}{\gamma_m}$.

Proof of $\frac{\gamma}{\gamma_m} = \frac{\alpha(1+n)(2+\theta)\gamma}{\beta-(2+\theta)(1-\tau)\bar{r}-1} + (1-\delta)$:

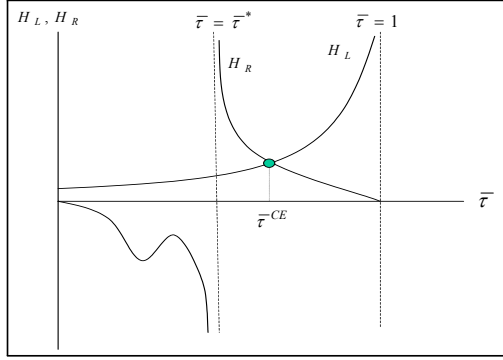
Substituting equations (1), (2), (8), (9), (10), and (11) in the good market clearing condition, (12), we obtain

$$k_{t+1}(1+n) = B_t k_t^\alpha x_t^v \left[\frac{\beta}{2+\theta} - v \frac{1-\tau_t}{\tau_t} \right].$$

Evaluating this expression on the balanced growth path and taking into account that $\gamma = \gamma_k = k_{t+1}/k_t$, we can write

$$B_t k_t^{\alpha-1} x_t^v = \frac{(1+n)\gamma}{\left[\frac{\beta}{2+\theta} - v \frac{1-\tau_t}{\tau_t} \right]}.$$

Figure 2: Proof of Proposition 1



On the balanced growth path, this expression must be equal to equation (19),

$$\frac{(1+n)\gamma}{\left[\frac{\beta}{2+\theta} - \nu \frac{1-\bar{\tau}}{\bar{\tau}}\right]} = \frac{\gamma_b \gamma_k^\alpha \gamma_x^{v-1} - (1-\delta)}{\alpha}.$$

Taking into account equation (20) and considering that $\gamma = \gamma_k$, and $\gamma_x = (1-\bar{\tau})/(1+n)$,

this equation can be expressed as

$$\frac{\alpha(1+n)(2+\theta)\gamma}{\beta - \nu(2+\theta)\frac{1-\bar{\tau}}{\bar{\tau}}} + (1+\delta) = \frac{(1+n)\gamma}{1-\bar{\tau}}. \quad \blacksquare$$

Proof of Proposition 1:

i) and ii) Substituting γ in the first equation of lemma 1:

$$(1+b)^{1/1-\alpha} \left(\frac{1-\bar{\tau}}{1+n} \right)^{\frac{\nu+\alpha-1}{1-\alpha}} - (1-\delta) = \frac{\alpha(2+\theta)(1+n)(1+b)^{1/1-\alpha} \left(\frac{1-\bar{\tau}}{1+n} \right)^{\nu/1-\alpha}}{\beta - \nu(2+\theta)\frac{1-\bar{\tau}}{\bar{\tau}}}. \quad (21)$$

This is a non linear equation for $\bar{\tau}$. Let H_R and H_L define the right and left hand sides of this equation, respectively. The proof lies in Figure 2, where $\bar{\tau}^* = \frac{(2+\theta)\nu}{\beta+(2+\theta)\nu}$. We will show that H_R and H_L are as in Figure 2 and, therefore, there exists a unique $\bar{\tau}^* < \bar{\tau} < 1$ that solves equation (21).

Since $-1 < \frac{\nu+\alpha-1}{1-\alpha} < 0$, the function H_L is defined $\forall \bar{\tau} \in [0, 1)$ and presents an asymptote

in $\bar{\tau} = 1$. Moreover, H_L is an increasing function $\forall \bar{\tau} \in [0, 1)$

$$\frac{\partial H_L}{\partial \bar{\tau}} = -\frac{\nu+\alpha-1}{1-\alpha} \left(\frac{1-\bar{\tau}}{1+n} \right)^{\frac{\nu+\alpha-1}{1-\alpha}-1} \frac{(1+b)^{1/1-\alpha}}{1+n} > 0.$$

Note that $H_L(0) = (1+b)^{1/1-\alpha} (1+n)^{\frac{1-\nu-\alpha}{1-\alpha}} - (1-\delta) > 0$.

On the other hand, H_R is defined $\forall \bar{\tau} \in [0, \bar{\tau}^*)$ and $(\bar{\tau}^*, 1]$. The function H_R presents an asymptote in $\bar{\tau} = \bar{\tau}^*$ such that

$$\lim_{\bar{\tau} \rightarrow \bar{\tau}^*(-)} H_R \rightarrow -\infty,$$

$$\lim_{\bar{\tau} \rightarrow \bar{\tau}^*(+)} H_R \rightarrow +\infty.$$

Given the characteristics of H_L and since $H_R(\bar{\tau}) \leq 0$, for all $\bar{\tau} \in [0, \bar{\tau}^*)$, then $\bar{\tau} \in [0, \bar{\tau}^*)$

such that $H_R = H_L$.

Moreover, H_R is a decreasing function $\forall \bar{\tau} \in (\bar{\tau}^*, 1]$,

$$\frac{\partial H_R}{\partial \bar{\tau}} = -\frac{\nu\alpha(2+\theta)(1+n)(1+b)^{\frac{1}{1-\alpha}} \left(\frac{1-\bar{\tau}}{1+n} \right)^{\frac{\nu}{1-\alpha}}}{\left[\beta - \nu(2+\theta)\frac{1-\bar{\tau}}{\bar{\tau}} \right]^2} \times \left\{ \frac{1+n}{(1-\alpha)(1-\bar{\tau})} \left[\beta - \nu(2+\theta)\frac{1-\bar{\tau}}{\bar{\tau}} \right] + \frac{2+\theta}{\bar{\tau}^2} \right\} < 0,$$

since $\left[\beta - \nu(2+\theta)\frac{1-\bar{\tau}}{\bar{\tau}} \right] > 0$ when $\bar{\tau} \in (\bar{\tau}^*, 1]$. This means that given the characteristics of H_L , there exists a unique $\bar{\tau} \in (\bar{\tau}^*, 1)$ such that $H_R = H_L$.

iii) Differentiating equation (21) with respect to all parameters it is easy to obtain

$$\frac{\partial \bar{\tau}}{\partial b} = \Psi \frac{1}{1-\alpha} \frac{1-\delta}{1+b} < 0,$$

$$\frac{\partial \bar{\tau}}{\partial \rho} = \Psi \frac{1-\delta}{1-\alpha} \ln(1+b) < 0,$$

$$\frac{\partial \bar{\tau}}{\partial \theta} = -\Psi \frac{\beta\alpha\gamma(1+n)}{\left[\beta - \nu(2+\theta)\frac{1-\bar{\tau}}{\bar{\tau}} \right]^2} > 0,$$

$$\frac{\partial \bar{\tau}}{\partial \delta} = \Psi < 0,$$

where $\Psi = \left[\frac{\partial H_R}{\partial \bar{\tau}} - \frac{\partial H_L}{\partial \bar{\tau}} \right]^{-1} < 0$.

On the other hand, lemma 1 states that $\gamma = (1+b)^{\rho/1-\alpha} \left(\frac{1-\bar{\tau}}{1+n} \right)^{v/1-\alpha}$. It is easy to calculate

$$\begin{aligned} \frac{\partial \gamma}{\partial b} &= \frac{\gamma}{1-\alpha} \left[\frac{\rho}{1+b} - \frac{\nu}{1-\bar{\tau}} \frac{\partial \bar{\tau}}{\partial b} \right] > 0, \\ \frac{\partial \gamma}{\partial \rho} &= \frac{\gamma}{1-\alpha} \left[\ln(1+b) - \frac{\nu}{1-\bar{\tau}} \frac{\partial \bar{\tau}}{\partial \rho} \right] > 0, \\ \frac{\partial \gamma}{\partial \theta} &= -\frac{\nu}{1-\alpha} \frac{\gamma}{1-\bar{\tau}} \frac{\partial \bar{\tau}}{\partial \theta} < 0, \\ \frac{\partial \gamma}{\partial \delta} &= -\frac{\nu}{1-\alpha} \frac{\gamma}{1-\bar{\tau}} \frac{\partial \bar{\tau}}{\partial \delta} > 0. \end{aligned} \quad \blacksquare$$

Proof of Lemma 2:

Any optimal equilibrium is a solution of the non-linear system (14)-(16) together with the restrictions of the planner's problem, (13).

Proof of $\tilde{\gamma}_k = \tilde{\gamma}_y = \tilde{\gamma}$:

Since the optimal condition for resource allocation, equation (16), is the same as the intertemporal resource allocation in the competitive equilibrium (equation (18)), we obtain

$$\tilde{\gamma}_k = \tilde{\gamma}_y = (1+b)^{1/1-\alpha} \left(\frac{1-\tilde{\tau}}{1+n} \right)^{v/1-\alpha} \equiv \tilde{\gamma},$$

in the same way that we have proved $\gamma_k = \gamma_y = \gamma$ in lemma (1). We will prove below that $\tilde{\tau} = R/(1+R)$ and therefore we have

$$\tilde{\gamma} = (1+b)^{1/1-\alpha} [(1+R)(1+n)]^{-v/1-\alpha}.$$

Proof of $\tilde{\gamma}_{c_1} = \tilde{\gamma}_{c_2} = \tilde{\gamma}_c$:

Taking the ratio of expression (14) in period $t+1$ and t ,

$$\frac{c_{2,t+1}}{c_{2,t}} \frac{c_{1,t}}{c_{1,t+1}} = 1.$$

And evaluating this on the optimal path with balanced growth,

$$\tilde{\gamma}_{c_1} = \tilde{\gamma}_{c_2} \equiv \tilde{\gamma}_c.$$

Proof of $\tilde{\gamma}_c = \tilde{\gamma}_k$:

The feasibility condition of the economy implies that production of the homogeneous good is the sum of consumption and investment in physical capital. In per worker terms,

$$y_{t+1} = c_{1,t+1} + \frac{c_{2,t+1}}{1+n} + (1+n) \tilde{\gamma}_k k_{t+1} - (1-\delta) k_{t+1}.$$

Reordering

$$\frac{y_{t+1} - c_{1,t+1} - \frac{c_{2,t+1}}{1+n}}{k_{t+1}} = (1+n) \tilde{\gamma}_k + (1-\delta).$$

Taking the ratio of this expression in period $t+1$ and t , and evaluating on the optimal path with balanced growth we get

$$\begin{aligned} \frac{y_{t+1} - c_{1,t+1} - \frac{c_{2,t+1}}{1+n}}{y_t - c_{1,t} - \frac{c_{2,t}}{1+n}} &= \frac{k_{t+1}}{k_t} = \tilde{\gamma}_k, \\ \tilde{\gamma}_y y_t - \tilde{\gamma}_{c_1} c_{1,t} - \tilde{\gamma}_{c_2} \frac{c_{2,t}}{1+n} &= \tilde{\gamma}_k \left(y_t - c_{1,t} - \frac{c_{2,t}}{1+n} \right). \end{aligned}$$

Since $\tilde{\gamma}_k = \tilde{\gamma}_y$, and $\tilde{\gamma}_{c_1} = \tilde{\gamma}_{c_2} = \tilde{\gamma}_c$, the consumption growth rate is given by $\tilde{\gamma}_c = \tilde{\gamma}_k$.

Proof of $\tilde{\gamma}_m = (1-\tilde{\tau})/(1+n)$:

Straightforward from valuation of the second restriction of the planner's problem on the optimal path with balanced growth.

Proof of $\tilde{\tau} = R/(1+R)$:

Substituting equation (14) and (16) into equation (15), we obtain

$$(1+n)(1+R) \frac{c_{1,t+1}}{c_{1,t}} = \frac{B_{t+1} k_{t+1}^\alpha x_{t+1}^{v-1}}{B_t k_t^\alpha x_t^{v-1}},$$

which evaluated on the optimal path with balanced growth, can be expressed as

$$(1+n)(1+R) \tilde{\gamma}_{c_1} = (\gamma_b)(\gamma_k)^\alpha (\gamma_x)^{v-1}.$$

It is easy to see from the production function that $\tilde{\gamma}_y = (\tilde{\gamma}_b) (\tilde{\gamma}_k)^\alpha (\tilde{\gamma}_x)^v$. Since $\tilde{\gamma}_y = \tilde{\gamma}_k$, this expression can be written as follows

$$(1+n)(1+R)\tilde{\gamma}_{c_1} = \frac{\tilde{\gamma}_k}{\tilde{\gamma}_m}.$$

Since $\tilde{\gamma}_m = (1 - \tilde{\tau}) / (1 + n)$, and $\tilde{\gamma}_k = \tilde{\gamma}_{c_1}$, we get $\tilde{\tau} = \frac{R}{1+R}$. ■

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